- Primary propulsion and attitude control of micro spacecraft.
- Precise positioning control of spacecraft constellations for interferometry missions.
- Potential gain in thrust-to-weight ratio

\[
\begin{align*}
\text{Thrust} & \propto P_c A_t \propto L_c^2 \\
\text{Weight} & \propto L_c^3 \\
\frac{\text{Thrust}}{\text{Weight}} & \propto L_c^{-1}
\end{align*}
\]

- For “Power MEMS” devices, typically in applications where batteries are currently used.
  - high power density

Pictures are from [http://www.onera.fr/conferences/micropropulsion/](http://www.onera.fr/conferences/micropropulsion/)
Characteristics and Challenges of Micro-Combustion System

- Power-generation devices currently developed are those that aim to generate power in the range of a few watts to milliwatts. The corresponding combustion devices are of the order of one centimeter in size.
- The characteristic length of micro combustors being developed to date, even in MEMS-sized systems, is sufficiently larger than the molecular mean-free paths of air and other gases flowing through the systems in which the physiochemical behavior of fluids is fundamentally the same as their macro-scale counterparts.
- As combustion volumes are reduced in size, issues of residence time, fluid mixing, thermal management, and wall quenching of gas-phase reactions become increasingly important.
- Surface-induced catalytic reactions is an attractive alternative in micro-systems.
For micro-devices with small characteristic lengths and consequently small Reynolds and Peclet numbers, the flow is primarily laminar, viscous effects and diffusive transport of mass and heat become increasingly important.

Low Reynolds number makes mixing of reactants a potential problem in micro-systems.

For diffusion flames, molecular diffusion is the rate-controlling process.

Since turbulence mixing is weak, species mixing is primarily through diffusion. Based on scaling analysis, the diffusion time and corresponding flame length is given by

\[ \tau_{\text{diff}} \sim \frac{d_m^2}{D} \quad L_f \sim U_0 \tau_{\text{diff}} \sim \frac{U_0 d_m^2}{D} \]

Complete and rapid mixing of adjacent laminar streams is desired, as is required for the initiation of a chemical reaction.

As the device scale is reduced, the increased surface-to-volume ratio results in a large heat loss to the chamber wall. Further, the temperature gradient within the solid wall decreases due to the reduced Biot number.
Combustion Issues

- For complete combustion, the flow residence time must be larger than the time required for chemical reactions. For non-premixed combustion, extra time and volume are needed for complete mixing.

- Flame quenching occurs if the total power generated inside the combustor is less than the loss to the wall

\[
W_{\text{tot}} < W_{\text{trans}}
\]

\[
W_{\text{tot}} = \rho_g \Delta H_r U_g \pi D L \sim \rho_g \Delta H_r U_g L^2
\]

\[
W_{\text{trans}} \sim \text{Pr}_g^{1/3} \left( \frac{P_g U_g}{R_g T_g} \right)^{1/2} k_f L^{3/2} T_g^{-1/2} (T_g - T_w)
\]

\[
P_g^{1/2} U_g^{1/2} L^{1/2} < \frac{\text{Pr}_g^{1/3} R_g^{1/2} k_f T_f^{1/2} (T_f - T_w)}{\mu_g^{1/2} \Delta H_r}
\]

- A higher chamber pressure and mass flow rate help prevent flames from extinction. An exceedingly high flow velocity, however, may lead to blowoff.
Development of Micro Power Generation Using Combustion

- Micro-scale power generation using combustion:
  - micro-combustors/reactors
  - micro turbines/engines
  - micro-rockets

MEMS-based gas turbine power generator developed at MIT

Meso- and micro-scale combustors developed at Penn State.

3-D Swiss-roll-type combustor-thermoelectric generator developed at USC
A scaled down version of a macroscopic whirl combustor (Yetter, Glassman & Gabler, 2000).
Made of Inconel with electro-discharge machining (EDM).
Combustor volume ranging from 10 to 108 mm$^3$.
Fuel injected perpendicularly to the tangentially injected oxidizer, and the flow exits the combustor tangentially.
Approximate flow residence time on the order of 0.1 to 1 ms for a total mass flow rate at around 0.02 g/s (evaluated at 1500K).
Theoretical Formulation

Full conservation equations

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} &= 0 \\
\frac{\partial \rho u_i}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} &= -\frac{\partial p}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} \\
\frac{\partial \rho E}{\partial t} + \frac{\partial \left[(\rho E + p)u_j\right]}{\partial x_j} &= -\frac{\partial q_i}{\partial x_i} + \frac{\partial (u_i \tau_{ij})}{\partial x_j} \\
\frac{\partial \rho Y_k}{\partial t} + \frac{\partial \rho Y_k u_j}{\partial x_j} &= \dot{\omega}_k - \frac{\partial \rho Y_k U_{k,j}}{\partial x_j}, \quad k = 1, \ldots, N
\end{align*}
\]

L-step reaction with N species

\[
\sum_{k_i}^{N} v_{ki}^\prime \chi_k \Leftrightarrow \sum_{k_i}^{N} v_{ki}^\prime \chi_k \quad \text{for} \quad i = 1, 2, \ldots, L
\]

\[
k_i(T) = A_i T^b \exp\left(-E_i / R_u T\right)
\]

\[
\dot{\omega}_k = MW_k \sum_{i=1}^{L} (v_{ki}^\prime - v_{ki}') \left[ k_{fi} \prod_{k=1}^{N} [\chi_k]^{v_{ki}} - k_{bi} \prod_{k=1}^{N} [\chi_k]^{v_{ki}'} \right] \quad \text{for} \quad k = 1, 2, \ldots, N
\]
Preconditioning method (Hsieh et al. 1997)

\[
\frac{\partial Q}{\partial t} + \frac{\partial (E - E_v)}{\partial x} + \frac{\partial (F - F_v)}{\partial y} + \frac{\partial (G - G_v)}{\partial z} = H
\]

\[
\bar{p} = p_0 + p_g
\]

\[
\Gamma \frac{\partial \hat{Q}}{\partial \tau} + \frac{\partial (E - E_v)}{\partial t} + \frac{\partial (F - F_v)}{\partial x} + \frac{\partial (G - G_v)}{\partial y} + \frac{\partial (G - G_v)}{\partial z} = 0
\]

\[
\hat{Q} = \left[ p_g, u, v, w, T, Y_1, Y_2, \ldots, Y_{N-1} \right]^T
\]

Finite volume approach

\[
\iiint_V \left( \Gamma \frac{\partial \hat{Q}}{\partial \tau} + \frac{\partial Q}{\partial t} \right) dV + \int_{S_x} \bar{W} \cdot \vec{n}_x dS_x + \int_{S_y} \bar{W} \cdot \vec{n}_y dS_y + \int_{S_z} \bar{W} \cdot \vec{n}_z dS_z = \iiint_V \bar{H} dV
\]
Behavior of Central Recirculation Zone (1/6)

Geometry of cylindrical combustor

![Diagram of cylindrical combustor with dimensions and streamlines]

Pseudo streamlines

Basic flame structure

One-step reversible reaction

\[ 2H_2 + O_2 \leftrightarrow 2H_2O \]

\[ \frac{d[c_{H_2}]}{dt} = -2\left[ k_f [c_{H_2}]^2[c_{O_2}] - k_b [c_{H_2O}]^2 \right] \]

\[ k_f = 1.102 \times 10^{19} \cdot \exp(-8025/T) \]
• generation of recirculation zone is caused by centrifugal effect.
• viscous effect at head end reduces the size of flow recirculation.

$U_{in,air} = 20 \text{ m/s}$
$U_{in,air} = 40 \text{ m/s}$
$U_{in,air} = 80 \text{ m/s}$
Both higher injection velocity and chamber pressure facilitate generation of central recirculation zone.

- Air
  - $P_0 = 1\text{ atm}$
    - $U_{in,\text{air}} = 20\text{ m/s}$
    - $U_{in,\text{air}} = 40\text{ m/s}$
    - $U_{in,\text{air}} = 80\text{ m/s}$

- H2
  - $P_0 = 2\text{ atm}$
    - $U_{in,\text{air}} = 20\text{ m/s}$
    - $U_{in,\text{air}} = 40\text{ m/s}$
    - $U_{in,\text{air}} = 80\text{ m/s}$
Behavior of Central Recirculation Zone (4/6)

• Fluid is transported downstream mainly in outer region.
• Tangential velocity is much higher in outer region.

Mass flux $\rho v_x$

Tangential velocity $v_\theta$

- $x=0.001\text{mm}$
- $x=0.003\text{mm}$
- $x=0.005\text{mm}$
• Flame structure is determined by the injection directions of fuel and air.
• Flame front is located where the fuel and oxidizer meet in stoichiometric proportions.

Iso-surface of temperature

$T = 1300K$

Iso-surface of mass fraction

$Y_{H_2} = 0.2$

$Y_{O_2} = 0.2$
Behavior of Central Recirculation Zone (6/6)

- The flame length increases with increasing injection velocity.
- The flame length decreases with increasing chamber pressure.

\[
P_0 = 1 \text{ atm}
\]

\[
U_{i,n,air} = 20 \text{ m/s} \quad U_{i,n,air} = 40 \text{ m/s} \quad U_{i,n,air} = 80 \text{ m/s}
\]

\[
P_0 = 2 \text{ atm}
\]

\[
U_{i,n,air} = 20 \text{ m/s} \quad U_{i,n,air} = 40 \text{ m/s} \quad U_{i,n,air} = 80 \text{ m/s}
\]
Combustion products are exhausted through a tangential square port.

- \( U_{in,air} = 100 \text{ m/s} \)
- \( P_c = 1 \text{ atm}, T_w = 800 \text{ K} \)
- \( \Phi = 1.0 \)

• The flow injected into the combustor is divided into three parts: main flow, upstream and downstream recirculating flows.
Combustion in Whirl Combustor (2/4)

- The small flow velocity in the recirculation region helps stabilize the flame in the upstream regime.
Distribution of temperature

- y = 0 mm
- z = 0 mm
- x = 0.2 mm
- x = 1.0 mm
- x = 2.0 mm
- x = 3.0 mm
Reactions occur in a limited regime near the head end.