System Performance and Thermodynamic Cycle Analysis of Airbreathing Pulse Detonation Engines

Yuhui Wu, Fuhua Ma, and Vigor Yang
Pennsylvania State University, University Park, Pennsylvania 16802

A modular approach to the study of system performance and thermodynamic cycle efficiency of airbreathing pulse detonation engines (PDEs) is described. Each module represents a specific component of the engine, and its dynamic behavior is formulated using conservation laws in either one or two spatial dimensions. A framework is established for assessing quantitatively the influence of all known processes on engine dynamics. Various loss mechanisms limiting the PDE performance are identified. As a specific example, a supersonic PDE for high-altitude applications is studied comprehensively. The effects of chamber configuration and operating sequence on the engine propulsive efficiency are examined. The results demonstrate the existence of an optimum cycle frequency and valve close-up time for achieving maximum performance in terms of thrust and specific impulse. Furthermore, a choked convergent–divergent nozzle is required to render the PDE competitive with other airbreathing propulsion systems, such as gas-turbine and ramjet engines.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cross-sectional area of detonation tube</td>
</tr>
<tr>
<td>$C_p$</td>
<td>constant-pressure specific heat</td>
</tr>
<tr>
<td>D</td>
<td>diameter of detonation tube</td>
</tr>
<tr>
<td>$D_e$</td>
<td>diameter of nozzle exit</td>
</tr>
<tr>
<td>$D_t$</td>
<td>diameter of nozzle throat</td>
</tr>
<tr>
<td>F</td>
<td>thrust</td>
</tr>
<tr>
<td>$F_{sp}$</td>
<td>specific thrust (air-based), $F/\dot{m}_a$</td>
</tr>
<tr>
<td>f</td>
<td>ratio of fuel to air mass flow rate</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$h$</td>
<td>flight altitude</td>
</tr>
<tr>
<td>I</td>
<td>impulse</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>specific impulse (fuel-based), $F/\dot{m}_f$</td>
</tr>
<tr>
<td>L</td>
<td>length of detonation tube</td>
</tr>
<tr>
<td>$L_{nozzle}$</td>
<td>length of nozzle section</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>$MR$</td>
<td>molar ratio of nitrogen to oxygen</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>mass flow rate</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>$p_p$</td>
<td>plateau pressure in single-pulse detonation study</td>
</tr>
<tr>
<td>q</td>
<td>heat addition per unit mass of air</td>
</tr>
<tr>
<td>$\dot{q}$</td>
<td>nondimensional heat addition, $q/C_pT_0$</td>
</tr>
<tr>
<td>$s$</td>
<td>entropy</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
</tr>
<tr>
<td>u</td>
<td>velocity</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>half conical angle of nozzle</td>
</tr>
<tr>
<td>$\beta$</td>
<td>ratio of nozzle length to detonation tube length</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of specific heat</td>
</tr>
<tr>
<td>$\eta_h$</td>
<td>thermodynamic cycle efficiency</td>
</tr>
<tr>
<td>$\tau$</td>
<td>time period</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>residence time of detonation wave, $L/\dot{u}_D$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>stoichiometric ratio</td>
</tr>
<tr>
<td>$\psi$</td>
<td>cycle static-temperature ratio, $T_1/T_0$</td>
</tr>
</tbody>
</table>

Subscripts

- $a$ = air
- close = time duration during which valve is closed
- cycle = pulse detonation engine operation cycle
- $D$ = detonation wave
- $f$ = fuel
- $i$ = preconditioned state
- purge = purging stage
- refill = refilling stage
- 0 = freestream condition
- 1 = fresh reactant upstream of detonation wave front
- 2 = combustion product downstream of detonation wave front
- 3 = flow property after isentropic expansion

I. Introduction

PULSE detonation engines (PDEs) have recently been recognized as a promising propulsion technology that offers advantages in thermodynamic cycle efficiency, hardware simplicity, operation scalability, and reliability. The potential for self-aspirating operation is highly attractive from the perspectives of efficiency and operation. Studies of PDEs have been conducted for several decades. The earliest experimental investigation may be traced back to Hoffman. Later performed a series of single-pulse detonation experiments with hydrogen/oxygen and acetylene/oxygen mixtures. Because a low-energy spark ignitor was used in their experiments and no deflagration-to-detonation (DDT) augmentation device was utilized, it is not clear whether full detonation waves were realized. Significant progress was made by Krzycki at the U.S. Naval Ordinance Test Station, demonstrating the use of propane/air mixtures for a pulse detonation device. The tube had an internal diameter of 1 in. (2.54 cm) and a length of 6 ft. (182.9 cm). Cycle frequency of up to 55 Hz was achieved using a high-energy spark discharge. Krzycki concluded that this intermittent detonation device was not promising for propulsion applications due to the low specific impulse associated with the limited cycle rates attained.

Exploratory research on detonation as an alternative reaction mechanism for airbreathing and rocket propulsion was terminated in the late 1960s due to the lack of funding and was not resumed until the 1980s. Helman et al. carried out a series of experiments with ethylene/oxygen and ethylene/air mixtures at the U.S. Naval Postgraduate School. Both single- and multicycle modes were studied. A predetonating ethylene/oxygen was employed to enhance the DDT process in the main tube filled with an ethylene/air mixture. The pressure recordings, however, suggested that full detonation
was not achieved, as revealed by the presence of significant compression waves preceding the pressure rise of the reported detonation waves.

Much effort has been applied to the study of various aspects of PDEs since the mid-1990s. Tables 1 and 2 summarize the experimental work performed to date. In single-pulse experiments, only detonation ignition, propagation, and attenuation were investigated at preconditioned states. Total impulse was obtained based on measured chamber pressure and/or force histories for a limited time period. In practice, negative thrust may appear due to the low-energy level of the gases in detonation tubes during the slowdown, purging, and refilling processes in a multicycle mode. As a result, system performance obtained from single-pulse experiments usually exceeded that in a real engine with a multicycle operation. In contrast, multicycle experiments involved all necessary PDE operation processes and, thus, provided more direct simulation. However, much important information required to characterize the system dynamics, such as airflow rate and purging/refilling data, was not measured in most experiments. The details for thrust measurements were not clearly defined either, rendering assessment of the performance of different systems a difficult task.

In parallel to experimental investigations, attempts were made both theoretically and numerically to estimate the performance of PDEs. Talley and Coy employed a lumped-parameter analysis to determine the theoretical limit of PDE performance by approximating the detonation chamber dynamics with an ideal constant-volume process. The blowdown time was assumed to be much longer than the characteristic wave transit times in the chamber. Tew developed a semi-analytical model for the impulse of a single-pulse detonation tube by means of dimensional analysis and empirical observations. In addition to these theoretical models, several numerical analyses based on one-dimensional approaches were carried out. One major deficiency of one-dimensional models is that the boundary condition at the detonation tube exit can not be correctly specified because it depends on the local flow evolution in the downstream ambient regime. Multidimensional simulations with computational

<table>
<thead>
<tr>
<th>Propellant</th>
<th>Reference</th>
<th>Configurations</th>
<th>$I_{dp}$, s, impulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$/O$_2$, 1 atm, 298 K, $\phi = 1.0$</td>
<td>Hinkey et al.(^6) (1995)</td>
<td>$L = unknown$, i.d. = 5.1 cm, fully filled, spark ignitor of 1.7 J, DDT enhanced by Schelkin spiral</td>
<td>Mixture-based, 240 s(^a), 185 s(^b)</td>
</tr>
<tr>
<td>H$_2$/air, 1 atm, 298 K, $\phi = 1.0$</td>
<td>Hinkey et al.(^6) (1995)</td>
<td>$L = unknown$, i.d. = 5.1 cm, fully filled, spark ignitor of 1.7 J, DDT enhanced by Schelkin spiral, predetonator filled with H$_2$/O$_2$ mixtures</td>
<td>Fuel-based, 1000 s(^c)</td>
</tr>
<tr>
<td>C$_2$H$_2$/O$_2$, 1 atm, 298 K, $\phi = 1.0$</td>
<td>Sanders et al.(^11) (2000)</td>
<td>$L = 135 cm$, i.d. = 3.8 cm, partially and fully filled, spark ignitor</td>
<td>Total impulse = 3000 N·s/m(^2), 792 s</td>
</tr>
<tr>
<td>C$_2$H$_2$/air, 1 atm, 298 K</td>
<td>Broda et al.(^15) (1999)</td>
<td>$L = 182.9 cm$, i.d. = 3.4 cm, $\phi = 1.1$–1.3, fully filled, spark ignitor of 3.5 J, DDT enhanced by obstacles</td>
<td>Mixture-based, 200 s(^d)</td>
</tr>
<tr>
<td>C$_2$H$_2$/O$_2$/N$_2$, $p$ and $T$ unknown</td>
<td>Sinibaldi et al.(^17) (2000)</td>
<td>$L = 190.5 cm$, i.d. = 5.7 cm, $\phi = 0.6$–2.0, fully filled, ignitor of 0.33–31 J, O$_2$/N$_2$ volumetric ratio: 100% and 75%</td>
<td>N/A</td>
</tr>
<tr>
<td>C$_2$H$_2$/O$_2$/N$_2$, 30–100 kPa, 298 K</td>
<td>Cooper et al.(^18) (2002)</td>
<td>$L = 101.6 cm$ (L/D = 13), i.d. = 7.6 cm, $\phi = 1.0$, fully filled, spark ignitor, N$_2$ dilution: 0–75% (by volume)</td>
<td>Mixture-based 170 s(^e) at $p = 100$ kPa</td>
</tr>
<tr>
<td>C$_2$H$_2$/O$_2$, 1 atm, 298 K</td>
<td>Sinibaldi et al.(^13) (2001)</td>
<td>$L = 120 cm$, i.d. = 12.7 cm, $\phi = 0.4$–1.0, predetonator, spark ignitor</td>
<td>N/A</td>
</tr>
<tr>
<td>C$_2$H$_2$/O$_2$/N$_2$, 0.5–1.0 atm, 298 K, $\phi = 1.0$</td>
<td>Cooper et al.(^18) (2002)</td>
<td>$L = 250 cm$, i.d. = 5.05 cm, $\phi = 1.0$, fully filled, spark plug of 0.05 J for DDT initiation, exploding wire ignitor of 203–502 J for direct initiation</td>
<td>Total impulse = 8.0 N·s(^f) (MR = 1.0)</td>
</tr>
<tr>
<td>C$_2$H$_2$/O$_2$/N$_2$, $\phi = 1.0$, 1 atm, $T$ unknown</td>
<td>Lieberman et al.(^20) (2002)</td>
<td>$L = 100 cm$, i.d. = 7.5 cm, N$_2$ dilution: 20–40% (by volume), driver section: C$_2$H$_2$/O$_2$ ($\phi = 1$) at $p = 1$, 4 atm</td>
<td>Mixture-based, 152 s(^g) (20% N$_2$ dilution)</td>
</tr>
</tbody>
</table>

\(^a\)Based on time history of pressure at thrust wall. \(^b\)Based on time history of force measured by load cell. \(^c\)Maximum impulse measured from ballistic pendulum displacement. |
Table 2 Survey of multicycle experimental investigation of PDEs

<table>
<thead>
<tr>
<th>Propellants</th>
<th>Reference</th>
<th>Configurations</th>
<th>$I_{sp}, s$ and Thrust, N</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$/O$_2$, 1 atm, 298 K, $\phi = 1.0$</td>
<td>Sterling et al.\textsuperscript{7} (1996)</td>
<td>Single tube, $f = 33$ Hz, i.d. = 2.2 cm, $L = 15.2$ cm</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Stuessy and Wilson\textsuperscript{1} (1996)</td>
<td>Single tube, $f = 10$–12 Hz, fully filled, air used as buffer gas</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cylindrical tube with i.d. = 7.6 cm, $L = 5.3$ cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Annular tube with i.d. = 2.5 cm, o.d. = 7.6 cm, $L = 53.3$ cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Annular tube with diverging conical nozzle</td>
<td></td>
</tr>
<tr>
<td>H$_2$/air, 1 atm, 298 K</td>
<td>Aarnio et al.\textsuperscript{22} (1996)</td>
<td>Single tube, $f = 5$ Hz, i.d. = 5.1 cm, $L = 121.9$ cm, $\phi = 1.0$</td>
<td>Fuel-based, 1333 s$^a$</td>
</tr>
<tr>
<td></td>
<td>Hinkey et al.\textsuperscript{3} (1997)</td>
<td>Single tube, $f = 10$ Hz, common air inlet manifold, i.d. = 5.1 cm, $L = 91.4$ cm, $\phi = 0.7$–1.3</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Schauer et al.\textsuperscript{24} (1999)</td>
<td>Multitube (1, 2, and 4), $f = 0.5$–100 Hz, i.d. = 5.1 cm, L = 91.4 cm and i.d. = 8.9 cm, $L = 91.4$ cm, air used as buffer gas, spark ignitor with DDT enhanced by Shchelkin spiral</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Schauer et al.\textsuperscript{25} (2000)</td>
<td>$f = 14$–40 Hz, $L = 91.4$ cm, i.d. = 5.1 cm, $\phi = 0.4$–2.85</td>
<td>Fuel-based for $\phi = 1.0$, $f = 16$ Hz, and 3.5 s ignition delay: 7100 s$^a$ (30% filling length) 4200 s$^a$ (90% filling length)</td>
</tr>
<tr>
<td></td>
<td>McManus et al.\textsuperscript{26} (2001)</td>
<td>Single tube, $f = 10$–35 Hz, conical converging nozzle, i.d. = 4.76 cm, $L = 25.4$ cm, $\phi = 0.6$, $T$</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Frankney et al.\textsuperscript{27} (2002)</td>
<td>Single tube, $f = 11$–21 Hz, converging nozzle, i.d. = 5.08 cm, $L = 182.88$ cm, $\phi = 1.0$, spark ignitor with DDT enhanced by Shchelkin spiral, air used as buffer gas</td>
<td>N/A</td>
</tr>
<tr>
<td>C$_2$H$_6$/O$_2$, 1 atm, 298 K, $\phi = 1.0$</td>
<td>Sterling et al.\textsuperscript{7} (1996)</td>
<td>Single tube, $f = 100$ Hz, $L = 50.8$ cm, i.d. = 2.2 cm, 1/3 tube filled, spark ignitor with DDT enhanced by unspecifed device</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Falempin et al.\textsuperscript{14} (2001)</td>
<td>Single tube, $f = 80$ Hz, i.d. = 5.0 cm, $L = 5.0$–42 cm, ignition source not mentioned</td>
<td>N/A</td>
</tr>
<tr>
<td>C$_2$H$_4$/air, 1 atm, 298 K</td>
<td>Broda et al.\textsuperscript{15} (1999)</td>
<td>Single tube, $f = 8$–10 Hz, $L = 182.9$ cm, i.d. = 3.4 cm, $\phi = 1.1$, spark ignitor of 4–8 J, DDT enhanced by obstacles</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Watts et al.\textsuperscript{16} (2000)</td>
<td>Single tube, $f = 10$ Hz, $\phi = 1.2$, spark plug of 25 J, DDT enhanced by obstacles, buffer air injected for 5–10 ms in each cycle</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Brophy et al.\textsuperscript{28} (2002)</td>
<td>Single tube, $f = 80$ Hz, i.d. = 4.0 cm, $L = 25$ cm, $\phi = 1.0$–1.8, spark ignitor with unknown energy</td>
<td>Total impulse, 0.48 N $\cdot$ s ($\phi = 1.0$)</td>
</tr>
<tr>
<td></td>
<td>Shimo et al.\textsuperscript{29} (2002)</td>
<td>$f = 15$ Hz, i.d. = 5.1 cm, $L = 82.2$ cm, $\phi = 0.9$, 60% filling ratio, spark ignitor with DDT enhanced by Shchelkin spirals</td>
<td>N/A</td>
</tr>
<tr>
<td>JP-10/O$_2$ $p$ and $T$ unknown</td>
<td>Brophy et al.\textsuperscript{30} (1998)</td>
<td>$f = 5$ Hz, $L = 15.2$, 30.5, 76.2 cm, i.d. = 3.8 cm, $\phi = 0.7$–1.7, ignitor of 1.4 J, ignition delay: 40, 50, and 70 ms</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Single tube, $f = 10$ Hz, i.d. = 3.81 cm, $L = 29$ cm, $\phi = 0.9$–1.2, ignitor of 0.5 J at locations $x = 0$–2D from head end</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Single tube, $f = 30$ Hz, i.d. = 4.0 cm, $L = 25$ cm, $\phi = 1.0$–1.8, spark ignitor with unknown energy</td>
<td>Total impulse, 0.59 N $\cdot$ s ($\phi = 1.0$)</td>
</tr>
<tr>
<td>C$_2$H$_6$/O$_2$/, $p$ and $T$ unknown</td>
<td>Farinaccio et al.\textsuperscript{32} (2002)</td>
<td>$f = 10$, 15, 20, 25 Hz, $L = 40$ cm, i.d. = 8.1 cm, $\phi = 1.0$, fully or partially filled, $N_2$ purged for 1/3-cycle period</td>
<td>Mixture-based, 71 s$^b$ ($f = 15$ Hz, fully filled)</td>
</tr>
</tbody>
</table>

*Based on time history of pressure at thrust wall. *Based on time history of force measured by load cell.

domains including both detonation tubes and ambient flows are, thus, required to describe the system dynamics faithfully, especially in the near field of the tube exit, where the flow is intrinsically multidimensional. To date, only single-pulse operations have been treated using two-dimensional analyses. Multicycle simulations have been limited to one-dimensional models, whose results are apparently questionable. Most of the analyses developed so far have considered only detonation chamber dynamics, not a complete PDE system.

The current work attempts to establish a global analysis to determine the overall system performance and thermodynamic cycle efficiency of airbreathing PDEs. Figure 1 shows schematically the configuration under consideration. It includes a coaxial supersonic inlet with mixed compression, a multitupe detonation chamber, and a nozzle. A rotary valve is placed in front of the combustor entrance to distribute the airflow evenly into the individual tubes. The following section describes the development of an ideal PDE thermodynamic cycle analysis to assess the theoretical limit of system performance. A modular approach is then utilized to analyze the engine dynamics numerically. As a specific example, an engine operating at a flight altitude of 9.3 km and a freestream Mach number of 2.1 is considered. The effects of various operating parameters on the engine propulsion efficiency are studied systematically. Finally, the influence of nozzle design is examined.

II. Thermodynamic Cycle Analysis

An ideal thermodynamic cycle analysis is presented in this section to estimate the theoretical limit of the performance of an airbreathing PDE. The work extends the approach of Heiser and Pratt\textsuperscript{16} for perfect gases with constant properties to accommodate property variations across the detonation wave front. Figure 2 shows the temperature–entropy diagram of an ideal PDE cycle. The corresponding Humphrey (constant-volume combustion) and Brayton (constant-pressure combustion) cycles are included for comparison. The process from point 0 to point 1 is an adiabatic, isentropic compression process, in the course of which the flow temperature is raised from its freestream value, $T_0$, to that at the combustor entrance, $T_1$. The path from point 1 to point 2 corresponds to the detonation process. Here the ZND model is adopted, that is, the dashed line from point 1 to point 1a corresponds to the entropy increase caused by shock compression and the solid line from point 1a to point 2 represents subsequent heat release due to chemical reactions. Point 2
Table 3 Variations of flow properties for stoichiometric H₂/air mixture

<table>
<thead>
<tr>
<th>Cycle</th>
<th>ρ₁ₘ / ρ₁</th>
<th>ρ₂ / ρ₁</th>
<th>T₁ₘ / T₁</th>
<th>T₂ / T₁</th>
<th>χ₁ₘ – χ₁</th>
<th>χ₂ – χ₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal PDE</td>
<td>15.50</td>
<td>5.47</td>
<td>9.82</td>
<td>1.04</td>
<td>1.84</td>
<td>1.16</td>
</tr>
<tr>
<td>Humphrey</td>
<td>N/A</td>
<td>N/A</td>
<td>9.17</td>
<td>N/A</td>
<td>1.89</td>
<td>1.17</td>
</tr>
<tr>
<td>Brayton</td>
<td>1.00</td>
<td>N/A</td>
<td>7.94</td>
<td>N/A</td>
<td>2.28</td>
<td>1.18</td>
</tr>
</tbody>
</table>

*a* T₁ = 300 K, ρ₁ = 1 atm, and γ₁ = 1.4.

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is known as the Chapman–Jouguet (C–J) point, where the chemical system reaches an equilibrium state and the detonation wave velocity and C–J properties can be easily determined by means of a chemical equilibrium analysis.

---

\[
\eta_{th, PDE} = 1 - \left[ \left( \frac{\gamma_1 - 1}{\gamma_2 - 1} \right) \left( \frac{\gamma_2}{\gamma_1} \right)^2 \left( \frac{1 + \gamma_1 M_0^2}{1 + \gamma_2} \right) \right]^{(\gamma_2 + 1)/\gamma_2} \times \psi^{1-[(\gamma_2 - 1)/(\gamma_2 - 1)](\gamma_2/\gamma_1) - 1} \left[ \frac{\dot{q}}{\dot{q}} \right] \]

(2)

where γ₁ and γ₂ are the specific-heat ratios of the unburned and burned gases separated by the detonation wave front, respectively. The Mach number of the detonation wave relative to the unburned gas, M₀, can be calculated using the following equation for fixed heat addition:

\[
M_0 = \left[ \frac{\gamma_2^2 - 1}{\gamma_1^2} + \frac{\gamma_2^2 - 1}{\dot{q}} \frac{1}{\gamma_1 - 1} \right] = \frac{\gamma_2^2}{\gamma_1} - 1 \frac{\dot{q}}{\gamma_1 - 1} \psi
\]

(3)

In comparison, the thermodynamic efficiencies of the Humphrey and Brayton cycles are

\[
\eta_{th, Humphrey} = 1 - \left[ \left( \frac{\gamma_2 - 1}{\gamma_1} \right) \left( \gamma_2 + 1 \right) \right]^{(\gamma_2 - 1)/\gamma_1} \times \psi^{1-[(\gamma_2 - 1)/(\gamma_2 - 1)](\gamma_2/\gamma_1) - 1} \left[ \frac{\dot{q}}{\dot{q}} \right]
\]

(4)

\[
\eta_{th, Brayton} = 1 - \left[ \left( \frac{\gamma_2}{\gamma_1} \right) \left( \gamma_2 - 1 \right) \left( \gamma_2 + 1 \right) \right]^{(\gamma_2 - 1)/\gamma_1} \times \psi^{1-[(\gamma_2 - 1)/(\gamma_2 - 1)](\gamma_2/\gamma_1) - 1} \left[ \frac{\dot{q}}{\dot{q}} \right]
\]

(5)

With the thermodynamic cycle efficiency available, the thrust \( F \) can be obtained by a control-volume analysis,

\[
F = (\dot{m}_f + \dot{m}_j)u_3 - \dot{m}_a u_0 = \dot{m}_f \left( \sqrt{u_0^2 + 2 \dot{m}_a q} - u_0 \right)
\]

(6)

where \( u_0 \) is the freestream velocity, \( \dot{m}_f \) the cycle-averaged air mass flow rate delivered to the engine through the inlet, and \( \dot{m}_j \) the cycle-averaged fuel mass flow rate. Note that the preceding analysis is based on the assumptions that every fluid particle experiences the same processes sequentially and that the effects of purging and bypass air are ignored. The fuel-based specific impulse can then be obtained as follows:

\[
I_{sp} = \frac{F}{\dot{m}_f g} = \frac{\sqrt{u_0^2 + 2 \dot{m}_a q} - u_0}{g}
\]

(7)

Figures 3 and 4 show a typical result of cycle efficiency and specific impulse for a stoichiometric hydrogen-air system.
The freestream velocity and temperature are $u_0 = 636 \text{ m/s}$ and $T_0 = 228 \text{ K}$, respectively. The nondimensional heat addition $q$ is 22.47. The system performance increases with increasing static temperature ratio. The PDE offers the best performance among the three cycles, especially when the static temperature ratio $\psi$ is smaller than 3. This may be because, for a given amount of heat addition, the Mach number of the detonation wave increases with decreasing $T_1$ (or $\psi$), as indicated by Eq. (3). The shock-compression effect becomes more significant for a lower $T_1$, leading to a higher increase in the temperature and pressure of the unburned gases before combustion. The $I_{sp}$ of an ideal PDE reaches 5263 s when $T_1 = 428 \text{ K}$, that is, $\psi = 1.877$.

### III. System Performance Analysis

This section deals with the development of a system performance analysis for airbreathing PDEs as shown schematically in Fig. 1. The study is based on a modular approach. Each module represents a specific component of the engine, and its dynamic behavior is formulated using complete conservation equations. The work involves the following three components: 1) supersonic inlet dynamics, 2) detonation chamber dynamics and system performance, and 3) effect of nozzle configuration. The effects of fuel supply, air distribution, and inlet isolator are ignored for simplicity. They can, however, be straightforwardly included as submodels in the overall engine performance analysis.

#### A. Supersonic Inlet Dynamics

The inlet and its interaction with combustor represent a crucial aspect in the development of any airbreathing engine, including PDEs. The inlet is designed to capture and supply stable airflow at a rate demanded by the combustor and to maintain high pressure recovery and stability margin at various engine operating conditions. The overall vehicle performance depends greatly on the energy level and flow quality of the incoming air. A small loss in inlet efficiency may translate to a substantial penalty in engine thrust. Moreover, any change in the inlet flow structure may modify the downstream combustion characteristics and subsequently lead to undesirable behaviors, such as flame blowoff and flashback. Thus, matching inlet behavior to engine requirements is of fundamental importance to designers.\(^{50}\)

In addition to its primary function of supplying air, an inlet has a determining influence on the dynamics of the entire system through its intrinsic unsteadiness and interactions with the combustion chamber. Typically, pressure waves are produced in the combustion chamber and propagate upstream to interact with the inlet flow through a manifold, where mixing of air and fuel occurs. The resultant flow oscillations in the inlet diffuser then either propagate downstream in the form of acoustic waves, or are convected downstream with the mean flow in the form of vorticity and entropy waves, and further reinforce the unsteady motions in the combustor. A feedback loop is, thus, established between the inlet and combustor.
B. Detonation Chamber Dynamics

The detonation chamber dynamics is formulated based on the conservation laws for a multicomponent chemically reacting system in two-dimensional coordinates. Diffusive transport is neglected in the current study because of its minor role in determining detonation dynamics and system performance. The governing equations and their associated boundary conditions are solved using a recently developed space-time conservation-element-solution-element method that circumvents the deficiencies of existing numerical methods for treating detonation waves and shock discontinuities. The resultant computer code is further parallelized using the message passing interface library with domain decomposition to improve its efficiency.

Both simple global and detailed chemical kinetics models are utilized. The former involves only one progress variable to characterize the chemical reaction rate and assumes constant properties. Because of its computational efficiency and reasonable accuracy in determining the PDE propulsive performance, the model is implemented in the present work. The associated thermochromical parameters are optimized by comparing the calculated detonation wave properties with those from the NASA chemical equilibrium analysis. The relative errors are less than 5% in terms of the detonation velocity and the C–J pressure and temperature. As part of the model validation effort, a series of single-pulse calculations were conducted for a straight tube of 60 cm in length initially filled with a stoichiometric mixture of hydrogen and air at preconditioned pressure $p_i$ and temperature $T_i$. A driver-gas region spanning 0.2 mm near the head end with a temperature of 2000 K and a pressure of 30 atm was employed to initiate the detonation wave directly. Four different numerical grids, with the sizes of 0.2, 0.1, 0.05, and 0.025 mm, were used to check the solution accuracy in terms of grid independence. All of the calculated pressure profiles collapsed onto a single curve, with the C–J properties matching the analytical values exactly. As a result, the 0.2-mm grid was chosen for the entire study to alleviate the computational burden. For a single-pulse operation, the head-end pressure remains at a plateau value $p_p$, that is, $p_3$ in Refs. 37 and 46, for certain period soon after the detonation initiation and then decays gradually to a level lower than the ambient state. The impulse can be determined by integrating temporally the force exerted on the head end from $t = 0$ to the instant when the head-end pressure reaches the ambient value. The contribution to the impulse from the ignition source is estimated to be less than 0.5%. Figure 7 shows the impulse per unit cross-sectional area as a function of the detonation residence time $\tau_D$, which is defined as the tube length $L$ divided by the detonation wave velocity $u_D$, i.e., $\tau_D = L/u_D$. Results can be correlated well in the following form:

$$I/A = 4.1(p_p - p_i)\tau_D$$

This expression is quite similar to those obtained from the semi-analytical analysis of Wintenberger et al. and the experimental work of Falempin et al. The constants of proportionality differ slightly in the various studies, for example, 4.13 in Ref. 37 for the stoichiometric $H_2/air$ mixture. The impulse is spread over the entire tube length, and the impulse per unit cross-sectional area is constant.

The tube is initially filled with a stoichiometric mixture of $H_2$ and air at ambient pressure and temperature. It takes about five cycles to reach steady cyclic operation. Figure 8 shows the $x-t$ diagram for the first cycle of operation, obtained by tracing the characteristic lines of the flowfield along the centerline of the tube. The time histories of the flow properties at the head end are also presented. The detonation wave is directly initiated by a hot driver gas and propagates downstream at the C–J velocity toward the unburned mixture (region 1). It then induces Taylor expansion waves (region 2) to satisfy the stationary condition at the head end, causing a uniform region (region 3) with constant-flow properties in the upstream.

The detonation wave reaches the reactant/air interface at the tube exit at $t = 0.305$ ms (point A), which deviates slightly from the following analytical prediction by 0.6% due to the effect of the externally imposed ignition source:

$$\tau_D = L/u_D = 0.6 \text{ m}/1956 \text{ m/s} = 0.307 \text{ ms}$$

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The wave then degenerates to a nonreactive shock, that is, the primary shock wave, proceeding farther downstream into the external region, followed by a contact surface separating the ambient air and combustion products. A sonic region is gradually formed near the tube exit due to the local flow expansion, as evidenced by the clustered characteristic lines in the $x-t$ diagram. Downstream of the sonic region, the flow is expanded to become supersonic and finally leads to the formation of a secondary shock to match with the subsonic flow behind the primary shock. This secondary shock wave

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Fig. 7 Generalized impulse curve for single-pulse detonation in straight tube with stoichiometric $H_2/air$ mixture.
moves farther downstream, meeting with expansion waves originating from the primary shock wave. These complicated flow structures can be also observed in Fig. 9, which shows the instantaneous pressure and density and their gradient fields at $t = 0.7$ ms. Many salient features are clearly shown, including the expansion fans, vortices, and rolled-up slip lines that are developed as the shock diffracts over the edge of the tube exit.

As the detonation wave catches the reactant/air interface and the resultant primary shock wave travels outside the tube, a series of expansion waves is generated, which propagate upstream, resulting in a nonsimple wave region (region 4) when interacting with the incoming Taylor waves. A simple wave region (region 5) is recovered after passing through the Taylor waves. The first expansion wave reaches the head end at $t = 0.935$ ms (point B), which can be determined by considering the interaction between the expansion and the Taylor waves and the sound speed in region 3. A similarity solution has been derived by Wintenberger et al.\textsuperscript{37} to predict this time instant analytically,

$$
    t = L / u_D + \alpha(L/c_s)
$$

(10)
where $\alpha$ is function of $\gamma$ and $M_D$ and can be calculated as

$$
\alpha = \frac{1}{2} \left( \frac{1}{M_D^2} \right)^2 \left\{ \frac{2}{\gamma + 1} \left( \frac{\gamma + 1}{\gamma} \right) (\gamma + 1)^2 - \frac{2}{\gamma + 1} \right\}^{-1}
$$

(11)

Application of Eqs. (10) and (11) gives rise to an analytical value of 0.958 ms. The slight difference between the numerical and the analytical solutions may be attributed to the numerical resolution and dissipation near the tube exit.

On the arrival of the first expansion wave at the head end, the pressure begins to decay gradually. These expansion waves reflect off the head end and form another series of expansion waves, further reducing the chamber pressure. The downstream-traveling expansion waves weaken the secondary shock and eventually cause it to move upstream.

The head-end pressure decays to 0.23 atm at $t = 2.4$ ms, at which point the purging stage begins. The head-end temperature is 1258 K at this instant. Because of the pressure difference across the entrance plane, a right-running shock wave is established, along with a series of central expansion waves and a contact surface between the burned gas and the cold air. Another contact surface forms between the fresh reactants and purging air when the refilling stage commences 0.1 ms later. The corresponding refilling pressure, velocity, and Mach number are about 0.91 atm, 500 m/s, and 1.2, respectively.

The time evolution of the pressure distribution along the centerline during the first cycle of operation is shown in Fig. 10.

The flow evolution during a steady operation cycle is examined. Figure 11 shows the $x$-$t$ diagram and time histories of flow properties at the head end for the fifth cycle. The main flow features remain qualitatively the same as those in the first cycle. However, the secondary shock wave disappears because the flow behind the primary shock wave is already supersonic. In addition, the head-end pressure and temperature begin to decay earlier relative to the first cycle, due to the rarefaction waves produced from the previous cycle. Also note that the detonation wave catches the leading fresh reactant at $x = 51.2$ cm instead of at the tube exit.

The impulse of each cycle is calculated by considering the momentum balance over a control volume enclosing the entire engine. The inlet flow loss is properly taken into account as detailed in Sec. III.A. The cycle-averaged specific thrust (air-based) and specific impulse (fuel based) are then obtained by dividing the impulse by the air mass and fuel weight for each cycle, respectively. For the baseline case, the fuel-based specific impulse is 2328 s. This may be compared to a ramjet engine operating at the same flight condition with perfect nozzle flow expansion, which has a specific impulse of about 460 s (Ref. 61). A parametric study was carried out to examine the effect of various operating times on the system performance. Figure 12 shows the result as a function of $\tau_{\text{close}}$.

The straight-tube system leads to a specific impulse far lower than its theoretical limit of 5263 s based on the thermodynamic cycle analysis for an ideal PDE, Eq. (7), which assumes isentropic flow processes in the inlet and nozzle. Although the calculated specific impulse can be improved by optimizing the operation frequency and timing, the net gain appears to be limited with the current design. Several fundamental mechanisms responsible for such an unacceptable performance have been identified. First, at high altitudes, the straight-tube design fails to preserve the chamber pressure during the refilling stage at a level sufficient to meet the requirements for the mass loading density of fresh reactants. Second, the low chamber pressure in the refilling stages causes a high-speed reactant stream in the tube and, subsequently, results in a large performance loss. It is well established that the stagnation pressure drop due to energy addition is proportional to the square of the Mach number. In the present case, the local Mach number may reach a value of up to 1.2 during the refilling process. The ensuing loss of thermodynamic efficiency becomes exceedingly large compared with conventional propulsion systems with subsonic combustion. Third, the lack of an appropriate flow expansion device downstream of the detonation tube gives rise to an extremely complicated flow structure near the tube exit. The internal energy of the exhaust flow can not be effectively converted to the kinetic energy for thrust generation, further deteriorating the situation.

C. Effect of Nozzle Configuration

In light of the limited performance of the straight-tube design, much effort was expended to study the effect of nozzle configuration on the system propulsive performance. The nozzle design for PDEs poses a serious challenge because of the intrinsically unsteady nature of the pulse detonation process. Recent studies based on single-pulse calculations and experiments indicate that the nozzle configuration may significantly change the thrust delivered by an engine. In addition to its influence on specific impulse through modification of the gas expansion process, the nozzle affects the chamber flow dynamics and, consequently, the timing of various phases of the engine operation cycle, especially for high-altitude and space applications.

The present work focuses on a choked convergent-divergent (C–D) nozzle because of its effectiveness in preserving the chamber pressure during the blowdown and refilling stages. In contrast, divergent and plug nozzles do not possess such an advantage, especially under high-altitude conditions, in spite of their superior performance for single-pulse operation at sea level. Figure 13 shows schematically the nozzle configuration considered herein, measuring a length of 20 cm. The slope angle is 45 deg for the convergent part and 15 deg for the divergent part. The ratio of the tube...
Fig. 11 Fifth cycle $x-t$ diagram and time histories of flow properties at head end under typical PDE operation: stoichiometric H$_2$/air mixture, $\tau_{cycle} = 3$ ms, $\tau_{close} = 2.4$ ms, and $\tau_{purge} = 0.1$ ms.

Fig. 12 Effect of valve close-up time on specific impulse, $\tau_{cycle} = 3$ ms and $\tau_{purge} = 0.1$ ms; straight tube with stoichiometric H$_2$/air mixture, $h = 9.3$ km, and $M_{\infty} = 2.1$.

Fig. 13 Configuration of single-tube PDE with C–D nozzle.

cross-sectional area to the nozzle throat area is 1.78, and the nozzle expansion ratio is 2.81. Several calculations were conducted for this configuration. The baseline case has $\tau_{cycle}$ of 3 ms and $\tau_{close}$ of 2.1 ms. The detonation tube is initially filled with a stoichiometric hydrogen/air mixture and the nozzle with air. The engine takes five cycles to reach steady operation. Figures 14 and 15 show the $x-t$ diagrams along the centerline of the tube and the time histories of flow properties at the head end for the first and eighth cycles of operation, respectively. The flow characteristics bear close resemblance to those of the straight-tube case. A major difference lies in the reflection of a shock wave from the convergent section of the nozzle, instead of expansion waves in a straight tube. The reflected shock then propagates upstream and causes an abrupt increase in pressure at the head end on its arrival, as evidenced in the pressure–time trace in Fig. 14. The nozzle throat remains choked during most of the cycle, thus helping preserve the chamber pressure. The pressure in the refilling stage is about 1.45 atm, which is substantially greater than the straight-tube case and, consequently, increases the mass loading density of fresh reactants. The relatively lower speed of the refilled mixture also enhances the system thermodynamic efficiency. The specific impulse of $\tau_{purge}$ s in the present case is 46% higher than the maximum specific impulse achieved by a straight tube, further demonstrating the effectiveness of a choked C–D nozzle in improving engine performance.

A parametric study is conducted to study the timing effect on system performance by varying $\tau_{cycle}$ and $\tau_{close}$. The purge time $\tau_{purge}$ is fixed at 0.1 ms. Figure 16 shows the effect of $\tau_{close}$ on the specific thrust $F_p$, defined as the cycle-averaged thrust per unit of air mass flow rate, and the fuel-based specific impulse $I_{sp}$ at four different cycle frequencies of 200, 250, 333, and 400 Hz. The corresponding cycle periods are 5, 4, 3, and 2.5 ms, respectively. When the straight-tube design is compared to the same operating condition, the present system with a choked C–D nozzle can indeed substantially improve the engine performance by a margin of 45%.

The specific thrust increases as $\tau_{close}$ decreases for all of the frequencies considered herein. This can be explained as follows. For a given $\tau_{cycle}$ and $\tau_{purge}$, a smaller $\tau_{close}$ translates to a shorter blowdown process. The resultant higher chamber pressure during the refilling stage increases the loading density of fresh reactants. The increased refilling period also enhances the amount of reactants delivered to the chamber. Combined, these two factors result in a higher cycle-averaged chamber pressure and, consequently, a higher specific thrust. Note, however, that the lower bound of $\tau_{close}$ is subject to three practical constraints. The first is concerned with inlet overpressurization. The head-end pressure must not exceed the stagnation pressure of the inlet air to allow for purging and refilling when the valve is open. The second is related to chamber overfilling. The fresh reactants should not flow out of the nozzle to the external region before being burned completely unless afterburning is considered.
Fig. 14  First cycle x–t diagram and time histories of flow properties at head end under typical PDE operation with C–D nozzle: stoichiometric H₂/air mixture, \( \tau_{\text{cycle}} = 3 \) ms, \( \tau_{\text{close}} = 2.1 \) ms, and \( \tau_{\text{purge}} = 0.1 \) ms; 1 = uniform unburned region, 2 = Taylor expansion waves, 3 = uniform region, 4 = non-simple wave region, and 5 = simple wave region.

Fig. 15  Eighth cycle x–t diagram and time histories of flow properties at head end under typical PDE operation with C–D nozzle: stoichiometric H₂/air mixture, \( \tau_{\text{cycle}} = 3 \) ms, \( \tau_{\text{close}} = 2.1 \) ms, and \( \tau_{\text{purge}} = 0.1 \) ms.
The third constraint, although commonly satisfied in practical cases, is that the valve close-up time should be sufficiently long to cover at least the time required for detonation initiation and propagation throughout the entire chamber. The upper bound of the valve close-up time (or the lower bound of the valve open time) is based on the requirement that an appropriate amount of fresh reactants be delivered to the chamber to produce thrust.

The effect of the valve close-up time on the fuel-based specific impulse follows the same trend as that of the air-based specific thrust, except for a small range of valve close-up below its lower bound. The specific impulse and specific thrust satisfy the following relation:

\[ I_{sp} = \frac{F_p (1 + \tau_{purgel/\tau_{edit}})}{fg} \]  

As the valve close-up time decreases, the factor \( (1 + \tau_{purgel}/\tau_{edit}) \) decreases and may override the increase of \( F_p \) consequently leading to a decrease in \( I_{sp} \) as shown in Fig. 16b.

For a given cycle period, the valve close-up time determines the filling length of fresh reactants. A larger valve close-up (or smaller valve open) leads to a smaller filling length in most cases and, consequently, decreases the specific impulse. This result, however, is in contrast to the previous experimental and numerical observations for single-pulse operations, which concluded that the specific impulse increases as the filling length decreases. One factor contributing to this discrepancy is that, in single-pulse studies, the pressure and temperature of reactants are preconditioned to ambient values, whereas in the present multicyle study the flow conditions of the refilled mixture depend on the timing of the engine operation. The use of a choked C–D nozzle also exerted a substantial influence on the chamber dynamics. Thus, significant differences exist between single-pulse and multicyle operations. The conclusions from single-pulse studies may not be applied to multicyle cases directly.

Figure 16 also demonstrates the existence of an optimum frequency for achieving a maximum performance for a given PDE configuration and flight condition. At a low cycle frequency, more reactants can be refilled into the detonation tube. As a consequence, a higher chamber pressure can be reached, and the engine efficiency improves. However, a large refilling time associated with low-frequency operation may cause chamber overfilling and, thus, degrade the performance. These two conflicting effects result in an optimum frequency. In the present study, the operating frequency of 250 Hz (\( \tau_{cycle} = 4 \) ms) offers the best performance. The highest specific impulse is 3676 s, slightly lower than its ramjet counterpart of 3866 s with optimum nozzle flow expansion.

**IV. Conclusions**

A comprehensive analysis has been established to study the system performance of airbreathing PDEs. The physical model of concern includes inlet, air-distribution unit, detonation tube, and nozzle. Results from parametric studies reveal that, for a fixed operating frequency, to increase in cycle period, leads to increased engine performance in terms of specific impulse and thrust. The straight-tube design gives rise to unacceptable performance, especially for high-altitude applications, due to its failure to preserve chamber pressure during the refilling stage. A choked C–D nozzle appears to be required to deliver performance at a level sufficient to compete with other airbreathing engines, such as ramjets. For a given engine configuration and flight condition, an optimum cycle frequency and valve close-up exist for achieving the best performance. A thermodynamic cycle analysis was also developed to determine the theoretical limit of the engine propulsive efficiency. Results were compared with those of the Humphrey and Brayton cycles.

For a typical supersonic mission with a flight Mach number of 2.1 and an altitude of 9.3 km, the maximum PDE specific impulse is 3676 s for a stoichiometric hydrogen/air mixture with proper account of the inlet and nozzle performance losses. This \( I_{sp} \) is lower than its ramjet counterpart of 3866 s with perfect nozzle flow expansion. Furthermore, the intrinsic unsteadiness, thrust vector variation, and other loss mechanisms not considered in the present analysis (such as the energy required for detonation initiation and flow losses associated with the inlet isolator, rotary valve, and air distributor) may render the PDE much less attractive. Further improvement and optimization of the system configuration and operation are required.

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**References**


